Maths Syllabus for PhD entrance test

Abstract Algebra:

Definition of Groups, Subgroups and Factor Groups, Lagrange's Theorem, Homomorphisms, Normal Subgroups. Quotients of Groups. Basic Examples of Groups including Symmetric Groups, and matrix Groups, Sylow's Theorems, Rings, ideals, prime and maximal ideals, quotient rings, Principle ideal domains, Polynomial rings, mod-p irreducibility test, Eisenstein's irreducibility criterion; Fields, Finite Fields.

Complex Analysis:

Algebra of complex numbers, the complex plane, polynomials, Power series, transcendental functions such as exponential, trigonometric and hyperbolic functions. Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, and Open mapping theorem. Taylor series, Laurent series, calculus of residues. Conformal mappings, Mobius transformations.

Functional Analysis:

Metric spaces, Normed linear spaces and Banach spaces, Bounded linear operators/functionals, Hahn-Banach theorem, open mapping and closed graph theorems, principle of uniform boundedness; Inner-product spaces, Hilbert spaces, orthonormal bases, projection theorem, Riesz representation theorem.

Linear Algebra:

Finite dimensional vector spaces over real or complex fields; Linear transformations and their matrix representations, rank and nullity; systems of linear equations, characteristic polynomial, eigen values and eigen vectors, diagonalization, minimal polynomial, Cayley-Hamilton Theorem, Finite dimensional inner product spaces, Gram-Schmidt orthonormalization process, symmetric, skew-symmetric, Hermitian, skew-Hermitian, normal, orthogonal and unitary matrices; Jordan canonical form; bilinear and quadratic forms.

Numerical Analysis:

Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

Ordinary Differential Equations:

Existence and Uniqueness of solutions of initial value problems for first-order ordinary differential equations, singular solutions of first-order ODEs, and system of first-order ODEs. General theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Sturm- Liouville boundary value problem, Green's function.

Partial Differential Equations:

Lagrange and Charpit methods for solving first-order PDEs, Cauchy problem for first order PDEs. Classification of second-order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

Real Analysis:

Limit of Functions. Continuous Functions, Continuity and Compactness, Continuity and Connectedness, Discontinuities, Monotonic Functions, Infinite Limits and Limit at Infinity. Derivative of a Real Function. Mean Value Theorem, Continuity of Derivatives, L'Hospital's Rule, Derivatives of Higher Order, Taylor's Theorem. Riemann integral, Uniform continuity of a function, sequence of functions, series of functions.

Statistics and Probability Theory:

Probability, Conditional Probability, Independent Events, Total Probability, and Baye's theorem. Random Variable, Probability Density Function, Distribution Function, Mathematical Expectation, Variance, Discrete Distributions: Binomial, Poisson, Continuous Distribution: Normal distribution. Central Limit Theorem, Joint Probability Distribution, Transformations of Random Variables, Confidence Intervals, Maximum Likelihood Estimators, Hypothesis Testing.

Topology:

Basic Concepts of Topology, Bases, Subspace Topology, Product Topology, Metric Topology, Connectedness, Compactness, Countability and Separation Axioms, Urysohn's Lemma.

1   Pattern of the Question Paper:

(1) Three 1-mark Question (either fill-in-the-blank or multiple-choice) from each section mentioned above.

(2) Either one 7-mark subjective type question or two subjective type questions (one worth 3 marks and the other worth 4 marks) from each section mentioned above.

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